Comment on "Role of heavy meson exchange in near threshold $NN \to d\pi$"*

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Abstract

In a recent paper by C. J. Horowitz (Phys. Rev. C **48**, 2920 (1993)) a heavy meson exchange is incorporated into threshold $NN \to d\pi$ to enhance the grossly underestimated cross section. However, that calculation uses an unjustified assumption on the initial and final momenta, which causes an overestimate of this effect by a factor of 3–4. I point out that the inclusion of the $\Delta(1232)$ isobar increases the cross section significantly even at threshold.

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A recent paper [1] proposes that heavy meson exchange (HME) involving a nucleon-antinucleon pair may be important in threshold pion production in the reaction $np \to d\pi^0$ and $pp \to d\pi^+$ (here generically included in the first reaction). This mechanism contributes to the two-nucleon axial charge [2,3], and so far has been the only way to explain the surprisingly large $pp \to pp\pi^\circ$ cross section at threshold [4]. The importance of this effect in this reaction is partly due to the absence of charge-exchange pion s-wave rescattering, dominant in the present $np \to d\pi^0$. A motivation for the inclusion of the HME mechanism to the deuteron reaction in Ref. [1] is the stated underestimation of the cross section [5] by theory almost by a factor of two. This addition to the conventional one-nucleon axial charge and s-wave pion rescattering doubles the s-wave cross section bringing the calculated results close to the data.

The aim of this Comment is to criticize an approximation used in Ref. [1], which exaggerates the HME effect in this reaction. The σ meson exchange leads to the operator

$$\mathcal{M}_{fi} \propto \frac{\sigma_i \cdot (\mathbf{p}' + \mathbf{p})}{2M} \frac{1}{M} \frac{g_\sigma^2}{m_\sigma^2 + \mathbf{k}^2} \tau_{i0} \tag{1}$$

for each nucleon i. Except for the momentum transfer dependent σ propagator this is similar to the Galilean invariance (axial charge) part of the direct production operator. Exchange of the other important ω meson has an additional part $\propto \boldsymbol{\sigma_1} \times \boldsymbol{\sigma_2}$, which changes the spin and does not contribute to s-wave production here. Eventually this operator leads to radial integrals (with an opposite sign convention from Ref. [1])

$$\begin{aligned} J_\sigma = \frac{g_\sigma^2}{4\pi} \int_0^\infty \Big[& (\frac{d}{dr} - \frac{1}{r}) v(r) j_0(\frac{qr}{2}) (\frac{e^{-m_\sigma r}}{2Mr}) u_{11}(r) \\ & - v(r)\, j_0(\frac{qr}{2}) (\frac{e^{-m_\sigma r}}{2Mr}) (\frac{d}{dr} + \frac{1}{r}) u_{11}(r) \Big]\, dr \end{aligned} \tag{2}$$

for the deuteron S-state and

$$\begin{aligned} J_\sigma = \frac{g_\sigma^2}{4\pi} \int_0^\infty \Big[& (\frac{d}{dr} + \frac{2}{r}) w(r)\, j_0(\frac{qr}{2}) (\frac{e^{-m_\sigma r}}{2Mr}) u_{11}^*(r) \\ & - w(r)\, j_0(\frac{qr}{2}) (\frac{e^{-m_\sigma r}}{2Mr}) (\frac{d}{dr} - \frac{2}{r}) u_{11}^*(r) \Big]\, dr. \end{aligned} \tag{3}$$

for the D-state, with the derivatives acting only on the nearest wave function. Similar equations are valid also for ω exchange.

Following Koltun and Reitan [6], Eqs. (25–30) of Ref. [1] seem to replace the momentum operator $(\mathbf{p}' + \mathbf{p})$ operating on both the initial and the final state wave functions with $2\mathbf{p}$, because the pion momentum does not significantly affect s-wave production at threshold. Although valid for the direct production part, this is no more allowed in the presence of the momentum transferring HME potential, which does not commute with the momentum operator. This assumption only picks (double) the latter terms in the above equations. Elimination of the derivative in the final state by integration by parts does not help, since a derivative of the potential emerges. The first line in Table I shows these integrals using $2\mathbf{p}$ for the momentum operator and agrees well with Ref. [1]. Correcting this approximation essentially halves the contribution from the deuteron S-state, since there the final state momentum contribution is small. However, for the D-state this is significant and in destructive interference with the main part, as can be seen from Table I. Overall the HME contribution to the amplitude is decreased by a factor of 4 and cannot account for the missing cross section. Instead of an increase of the conventional α by 86 μb reported in Ref. [1], the change is now 18 μb. In these calculations the Bonn potential A(R) σ and ω couplings and form factors are used with Reid soft core wave functions as in Ref. [1].

The same approximation is used also for $pp \to pp\pi^\circ$ in Ref. [3]. Therefore, as a check, it was established that in $pp \to pp\pi^0$ the σ and ω exchanges alone give a good description of the low energy data [4]. With the more precise treatment of the final state momentum, however, the σ contribution decreases to nearly a half, while the ω effect is enhanced enough to compensate this loss. It may be noted that the σ and ω mesons were by far the most important in Ref. [3].

However, the present reaction is more involved than is apparent from the above discussion. The threshold description of Koltun and Reitan [6], employed in Ref. [1] with modern two-nucleon potentials and deuteron wave functions does not consider the role of an explicit virtual $\Delta(1232)$ isobar excitation in producing pions. This dominates p-wave production.

Although the centrifugal barrier in the P-wave baryon states suppresses the Δ components to some extent, it can be seen from Fig. 1 that even for threshold s-wave pions the isobar effect is by no means negligible and its inclusion triples the cross section. Therefore, in a more complete model the threshold cross section may be actually *overestimated* even without HME. By far most of this increase in s-wave pion production comes from the normal elementary (p-wave) emission of the pion from the Δ followed by s-wave rescattering from the second nucleon. Adding HME slightly increases the overestimation as shown in Fig. 1.

The present calculation treats the Δ isobar on the same basis as the nucleons by a system of coupled Schrödinger equations for a modified Reid soft core potential with $N\Delta$ admixtures in the initial NN state [7]. This causes also some short range changes in the NN wave function reflected in HME as seen in the third line of Table I. HME is included only in the nucleon sector. Energy dependence is allowed for s-wave pion rescattering to fit on-shell πN scattering [8], but except for the virtual $N\Delta$ admixtures the model reduces to the formalism of Koltun and Reitan at threshold. (A monopole form factor with $\Lambda = 700$ MeV is also included to account for off-shell rescattering.)

Another aspect for caution in adding new mechanisms to threshold amplitudes is the changes caused in observables at higher energies, where there are much more data to basically fix the amplitudes. The analyzing power A_y between 500 and 600 MeV is particularly sensitive to the s-wave pion amplitude. In this region the coupled-channels model used above to produce the solid curves has been particularly successful. The use of a smaller s-wave amplitude to fit the threshold cross section would produce too high an analyzing power, whereas a larger one would yield too deep a minimum in it. Again it is lucky that the HME effect is small in this reaction. Further, since the low energy analyzing power data [9] can be easily fitted by simply scaling the s-wave amplitude with the factor $\sqrt{\sigma(\mathrm{th})/\sigma(\exp)}$, which compensates for the overestimation of the s-wave amplitude, one may conclude that apparently the p-wave amplitude is under control also close to threshold.

The overestimation of the threshold cross section by the full model, which agrees well with the data in the Δ region, poses a slight problem of detail indicating that either the

energy dependence of off-shell pion rescattering is not properly incorporated or some physical mechanism is still missing in the present models of pion production. However, HME does not appear important here, whereas a significant contribution from the Δ is likely to survive improvements of the model.

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FIGURES

FIG. 1. Low-energy $pp \to d\pi^+$ cross section divided by $\eta = q_\pi/m_\pi$. The solid curves show the starting point before the addition of HME with the Δ included in all partial waves (the lower one is the s-wave contribution), while the dashed curve is the s-wave contribution without the Δ. The dotted and dash-dot curves have also the HME added to these calculations of the s-wave. The data are from Ref. [5].

TABLES

TABLE I. Integrals of Eqs. (2,3) (in $\mathrm{fm}^{-1/2}$) for σ and ω exchanges and S and D final states for $\eta = q_\pi/m_\pi = 0.1424$. The total has also a factor $1/\sqrt{2}$ multiplying the D state as required by angular momentum algebra [6].

Model	σ, S	σ, D	ω, S	ω, D	Total
$2\mathbf{p}$	-0.0700	-0.0519	-0.0004	-0.0002	-0.1223
$\mathbf{p}' + \mathbf{p}$	-0.0284	-0.0202	0.0162	0.0119	-0.0287
$N\Delta$	-0.0260	-0.0181	0.0059	0.0037	-0.0373